

STEM FORM OF WHITE OAK

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HISTORICAL DEVELOPMENT OF FORM STUDIES

METHODS of determining stem volumes of standing trees have been and still are of absorbing interest to foresters. Estimates of stem contents are required each time a forester takes an inventory of his working stock. Consequently, any device that will give accurate volumes with a minimum expenditure of labor is eagerly welcomed.

One of the earliest of these devices to come into common use was the volume table, which was suggested by H. Cotta in his work, *Systematische Anleitung zur Taxation der Waldungen* (Vol. 3, p. 6). In the early volume tables the assumption was made that trees of the same diameter and height possessed the same stem volume.

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Contemporaneously with the origin of the conception of volume tables Oberforster Joh. Christ. Paulsen in 1800 introduced into a manuscript¹ the theory of form variation among trees of the same diameter and height. He expressed this variation as a reduction factor whereby the volume of a cylinder of given dimensions was reduced a certain amount to determine the volume of a tree of the same dimensions and of a given form (18, p. 202). Paulsen recognized three form factors, 0.75, 0.66 and 0.50, and he assigned trees to one or another of these three form classes on the basis of crown length.

Hossfeld, in 1812, took a step forward in the study of stem form when he determined a formula by which the reduction factor could be calculated.

Schwappach (28, p. 48) distinguished three varieties of reduction factors, or form factors, as they came to be called:

(1) The breast-high form factor he defines as the ratio between the volume of a tree stem and the volume of a cylinder of equal height and diameter at breast height. This form factor introduces error, since the actual height at which the base diameter is measured remains at a constant height of 1.3 meters above the ground for all trees. On trees that are short this may be one third of the total height, whereas on tall trees this may be no more than one thirtieth of the total height.

(2) The normal form factor overcomes this weakness by making the height at which the base diameter is measured a constant percentage of the total height. The normal form factor is defined as the ratio between the volume of a tree and the volume of a cylinder of equal height and diameter at a height of $\frac{1}{n}$ times the total height of the tree. In practice Schwappach states that n is most frequently given a value of twenty.

(3) The Riniker form factor, introduced by Riniker in 1873, is the ratio between the volume of the portion of a tree lying above the breast-high point and the volume of a cylinder having

¹ This manuscript was published in 1845 in Hundeshagen-Klauprecht's *Beiträge zur Forstwissenschaft*.

a diameter equal to the breast-high diameter of the tree and a height equal to the length of the tree portion above breast height.

Schuberg (26) found in 1891 that the ratio between the diameter at the middle of the stem and the breast-high diameter could be used to determine corrections that should be applied to silver-fir volume tables in order to take form into consideration and to get more accurate volumes.

Almost simultaneously Kunze (14) proposed a formula for the calculation of the form factor. By this formula the form factor

$$f = \frac{d_{\frac{1}{2}}}{D} - c. \quad (1)$$

In this equation $d_{\frac{1}{2}}$ is the diameter at half the height of the tree, D is the diameter at breast height, and c is a constant that varies for different species. Kunze found the value of c to be about 0.21 for Norway spruce (*Picea Abies* Karst.) and about 0.20 for Scotch pine (*Pinus sylvestris* L.).²

In 1899 Schiffel (25) tested further the theory of Schuberg and concluded that as a form-determining factor the ratio between the diameter at mid-height and breast height was very satisfactory. This ratio Schiffel called the "form quotient," and grouped trees into form classes on the basis of the form quotient. Developing still further the formula of Kunze, Schiffel later arrived at a formula for tree volume based upon height, diameter and form.

$$V = h (0.16D + 0.66d_{\frac{1}{2}}). \quad (2)$$

In this formula V indicates stem volume, h is the height, D , the diameter at breast height, and $d_{\frac{1}{2}}$, the diameter at mid-height.

If any form factor is viewed from a purely theoretical standpoint it may be considered as the ratio between the volume of a solid generated by the rotation about the x -axis of a curve of the type

$$y^2 = px^r \quad (3)$$

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² See 28, p. 55.

a percentage distance of x from the tip and the base diameter. The nature of the rotation solid will depend upon the magnitude of r . When $r = 1$ the solid generated will be a paraboloid. A right circular cone will be generated when $r = 2$, and a neiloid when $r = 3$. It can be shown that the theoretical form factor³

$$f = \frac{1}{r+1} \left(\frac{l}{l-a} \right)^r. \quad (4)$$

Here r is the exponent of x in equation (3), l is the total height of the tree, and a is the distance from the ground to the point where the base diameter is measured.

In 1910 Jonson (9) made use of the logarithmic equation of Höjer to express the taper of Norway spruce stems:

$$\frac{d}{D} = C \log \frac{c+l}{c}. \quad (5)$$

In equation (5) d is the diameter at any point on the stem, D is the base diameter, l is the distance from the tip to the point at which the diameter d is measured, and C and c are constants that vary with the form class. Jonson computed the $\frac{d}{D}$ relationship at each tenth of the stem length above breast height for the sample Norway spruce trees with which he was working, and concluded that, if trees were grouped into classes on the basis of the form quotient, all trees falling into the same form class would exhibit similar form. This means that large trees and small trees are built according to the same pattern, and that the index to the form of any tree stem is the form quotient.

He arbitrarily selected form classes five units in width, the class mark⁴ being 0.55, 0.60, 0.65, 0.70. For each form class he computed the constants C and c in Höjer's equation.⁵ He chose these constants for each form class in such a way that the $\frac{d_{\frac{1}{2}}}{D}$ ratio gave the class mark for that form class. Having determined

³ For a complete development of this formula see Appendix A, p. 324.

⁴ For a definition of class mark see p. 287.

⁵ The details of the procedure used in determining the constants in Höjer's equation are given in Appendix B, pp. 327-328.

the constants for the several form classes, Jonson substituted those values for l in equation (5) that would give a diameter ratio at each tenth of the stem above breast height. In this manner a set of theoretical diameter ratios was constructed for each form class. Jonson divided his sample trees into form classes 0.60, 0.70, 0.80, and compared the actual diameter ratios determined by measurements on the tree stems with the theoretical ratios calculated by means of Höjer's equation. He found a very satisfactory comparison, and concluded that the equation of Höjer could be used to represent the form of Norway spruce.

In a manner similar to the one used to calculate formulae for the theoretical volume and form factor of a paraboloid,⁶ Jonson calculated formulae for the theoretical volume and form factor of a body generated by the rotation of Höjer's curve about its l -axis. Later he prepared tables that give tree volumes for trees of various diameters, heights and form classes (12).

Jonson also calculated diameter ratios from actual diameter measurements on Scotch pine. These measurements were taken at each tenth of the stem length above breast height. When he compared these ratios with the theoretical ratios given by Höjer's equation he found an appreciable disagreement in the upper portion of the stem (10, p. 291). However, by the addition of a "biologic" constant to the equation he secured a good agreement with the observed diameter ratios. The modified equation is

$$\frac{d}{D} = C \log \frac{c + l - 2.5}{c}. \quad (6)$$

The symbols used in this equation are the same as those that were used in equation (5).

Jonson realized that in order to make his application of Höjer's equation practicable he must devise some way to determine in the woods the form quotient of any tree. Using the theory of Metzger, which posits that the bole of a tree is constructed in a form that will best withstand the pressure of the wind against the tree crown, he determined the form class to which a tree belongs by the height of the form point (11, p. 244). According to

⁶ See Appendix A, p. 324.

Jonson, the form point height is the height of a paraboloidal girder which offers the same resistance to bending that is offered by a tree of similar diameter, and form in the lower portion. The girder is formed by the rotation of the parabola

$$\frac{d}{D} = \sqrt[3]{\frac{l}{L}}. \quad (7)$$

In this equation L and l are distances from the top of the paraboloid at which D and d are measured.

Jonson determined the theoretical form point height for form class 0.70 in the following manner: Theoretical diameter ratios computed by the use of Höjer's equation for form class 0.70 show that at two tenths of the height of the stem above breast height the theoretical diameter ratio is 39.95, and at breast height the ratio is 100. To determine the equation of a cubic paraboloid whose height is x , whose base diameter ratio is 100 and whose diameter ratio at $x=20$ is 39.95, substitution of these known values is made in equation (7), giving

$$\begin{aligned} \frac{39.95}{100} &= \sqrt[3]{\frac{x-20}{x}} \\ 0.8995^3 &= \frac{x-20}{x} \\ x &= \frac{20}{1 - (0.8995)^3} \\ x &= 73.5. \end{aligned}$$

The heights of the girder for other form classes were obtained by determining from Höjer's equation the theoretical diameter ratio at two tenths of the height of the stem above breast height. This ratio was substituted in equation (7), and x was computed as for form class 0.70. Since the diameters of the girder and the curve rotation body agree very closely in the lower portions, the height at two tenths above breast height is very probably an arbitrary one.

Jonson also developed an empirical equation by means of which the form point height may be computed for any form class (11, p. 255):

$$F_p = \left(q - 0.081 + \frac{0.07^{(7)}}{h - 1.3} \right)^2 \times 1.896. \quad (8)$$

In this equation F_p is the percentage form point height, h the total height of the stem in meters, and q the form quotient inside bark. Equation (8) may be rewritten as follows:

$$q^{(8)} = 0.726 \sqrt{F_p} + 0.081 - \frac{0.07}{h - 1.3}. \quad (9)$$

From the equation in this form the form quotient inside bark can be calculated when the form point is known.

Figure 5 presents diagrammatically the paraboloidal girder with reference to the tree stem and shows the girder length to be about 73.5 per cent of the stem length. It also shows that in the lower portion of the stem the girder and stem have very nearly the same diameter ratios.

The tip of the girder is considered to be the point at which the force of the wind against the crown is concentrated. For practical determination in the woods the height of the form point is considered the geometric center of the outline of the crown, and the height of this point is measured with the aid of a Christen hypsometer. Since the

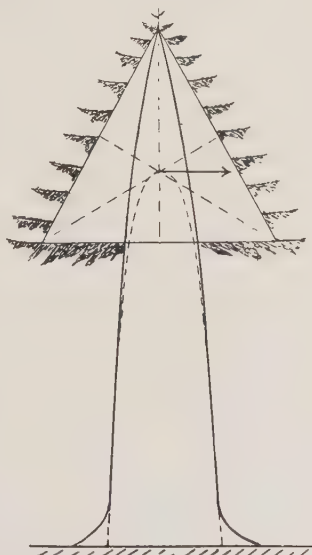


FIG. 5. A tree stem of form class 0.70 containing a theoretical girder of equal length (from Jonson)

⁷ In the original publication 0.07 is erroneously given as 0.70. See 13, p. 184.

⁸ If the form class outside bark is desired for thick-barked trees this expression must be corrected by a factor of the type $\frac{1.0p_1}{1.0p_0}$, where p_1 indicates the double bark as a percentage of the diameter inside bark at the middle of the stem, and p_0 the same percentage at breast height.

outline of the crown is frequently very irregular, this determination can be only approximately correct.

Petrini (21, p. 170) has investigated the accuracy with which the form point can be determined by different estimators. His conclusions are that different estimators arrive at different average form points for any given stand, but that the standard deviation of such averages is ± 1.17 form-point units⁹ from the mean of the average form points of all estimators. The average form points obtained by different estimators may differ somewhat more from one another. In this case Petrini found the standard deviation to be ± 2.0 form-point units. He also concluded that for a single tree the standard deviation of the estimates of form point by several estimators was ± 3.1 form-point units.

Tirén (29, p. 391) advanced the following equation for determining the diameter ratio at any height:

$$\frac{d}{D} = P \log \left(\frac{x + \sqrt{k^2 + x^2}}{k} \right). \quad (10)$$

In this equation d and D are upper and breast-height diameters respectively, P and k are constants, and x is the percentage distance from the tip.

Petterson (24, p. 39) proposed a modification of Höjer's equation in which he used only one curve of the form

$$y = \log x^{(10)} \quad (11)$$

instead of a series of curves, each of which has a different set of values for C and c .¹¹ In equation (11) y is the $\frac{d}{D}$ diameter ratio where d is measured at any percentage distance, x , from the tip. The curve of this equation is shown in Figure 6.

The units are seen to be the same for both the x - and y -axis. When $y = 0$, $x = 1$, and for values of x smaller than 1, y is negative. The portion of the curve above the x -axis is assumed by

⁹ A form point unit is defined as 1 per cent of the height of the tree.

¹⁰ For a development of the equation of Petterson see Appendix C, p. 328.

¹¹ See Appendix B, p. 327.

Petterson to represent the form of the tree stem. The tip is always at the point $x = 1$, but the base of the tree falls at varying values of x . As the base of the tree approaches the value $x = 1$, the

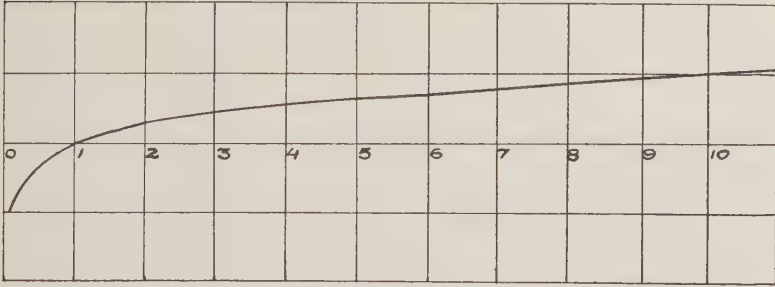


FIG. 6. Curve of equation $y = \log x$

form quotient becomes smaller, until when the base reaches the point $x = 1$ the form quotient reaches the limiting value of 0.50.

Behre (1, p. 681) has proposed an hyperbolic curve¹² for the expression of stem form:

$$y = \frac{x}{a + bx}. \quad (12)$$

In this equation y is the ratio between any given diameter and the base diameter, x is the percentage distance of the given diameter from the tip to breast height, and a and b are constants that vary with the form class. The constants in Behre's equation are very easily computed; their algebraic sum is always unity.

Jonson (13, p. 172) compares the diameter ratios as they are computed from the formulae of Höjer, Höjer-Jonson, Tirén, Behre and Petterson. His summary shows that the differences between the diameter ratios as given by the several formulae do not amount to 1 per cent in the lower 60 per cent of the stem length. Above that height variations as great as 4 per cent occur. Höjer's unmodified equation gives the highest ratios, but the other

¹² Langsaeter (15, p. 141) has expressed Behre's equation in the form $y = 1 - \frac{1}{x}$.

four equations agree rather well among themselves. The conclusion is that the results are very nearly the same if any of the last four equations is used.

Very recently Heijbel (8, p. 144) has attempted to show that no single curve can accurately portray the stem form of Scotch pine. He has fitted three separate curves to different portions of the stem. His investigation shows that the portion of the stem from the root swell to the crown is best represented by a tangent equation.¹³ The deviations in the crown portion of the stem are sufficient to cause Heijbel to compute a correction formula, which gives the correction that should be applied to the tangent curve in order to secure better agreement with actual tree form in the portions of the stem lying within the crown. Heijbel believes that the trunk form from the stump to 10 per cent of the height is best represented by a power equation, and accordingly develops one that represents the taper of Scotch pine in this portion of the stem.

From this brief historical review it is seen that European investigators have for many years pointed out the importance of stem taper in accurate tree volume determination. In American forest practice, on the contrary, the volume tables most commonly used are based upon diameter and height only. There are, however, variations other than the taper in the upper portion of the stem that influence the accuracy of American volume tables.

Unusual variations in diameter at breast height are of greater significance than variations elsewhere on the stem, for, according to the present practice, tables of volumes of trees of given heights and breast-high diameters show the same volumes for two trees if they have the same heights and diameters. Root swell, however, if it extends above breast height, might easily add one inch to the breast-high diameter of one tree and not influence the diameter of the other appreciably. The first tree, therefore, is in a diameter class that is one inch greater than the diameters of the upper sections warrant, and the tabular volume is in excess of the true volume.

¹³ For a development of the formulae of Heijbel see Appendix D, p. 331.

Since foresters are primarily interested in the wood content beneath the bark,¹⁴ the influence of bark thickness upon the measured diameters outside the bark is also important. Especially is this true of the breast-high measuring point. Measurements on standing trees are customarily made on the bark, and an accurate conception of the thickness of bark is essential to satisfactory determinations of wood content.

Form studies are, therefore, of significance, since through them foresters arrive at systems of tree-volume determination that permit greater accuracy than could possibly be obtained if form were disregarded. The rate of taper of the main stem and the effect of root swell and bark thickness on the principal diameter measurement are the chief factors that cause variations in volumes of trees having the same height and diameter at breast height.

The work done heretofore to develop mathematical equations as expressions of tree form has been confined almost entirely to coniferous species. For these species the form-class method has seemed to be very helpful in increasing the accuracy of wood-volume determination. Foresters have been seeking some method of caring for form in the broad-leaved species and have hoped that the form-class method might apply to them. The characteristic habit of growth of conifers gives them a regular, consistent taper, whereas most hardwoods have no such regularity in form. Large branches occurring irregularly introduce this variability.

Hawley and Wheaton (6, p. 7) have made use of the form-class method in their study of New England hardwoods. Hedeby (7, p. 193) began a comprehensive study of the form of oak¹⁵ in Sweden; he recently published a preliminary report upon his researches. In this report he considered only oak grown in a relatively closed forest condition and under intensive forest management. As a result, his basic material is more regular in

¹⁴ For particular species, such as hemlock and oak, foresters may be interested in determining annual increments and total yield of bark, as well as of wood, since the bark value for tannin is a matter of importance.

¹⁵ Two species of oak are indigenous to Sweden, *Quercus pedunculata* Ehrh. and *Quercus sessiliflora* Sm.

its taper than if he had considered also trees from open stands. From a meager sample of fifty-five closed-stand oaks he concluded that Höjer's equation, modified by the addition of a certain "biologic" constant, fits the oak data relatively well. For open-stand oak he offers no solution in his preliminary work.

Second-growth stands of hardwoods in the eastern United States do not generally fall into the class of material upon which Hedeby made his preliminary report. Our stands are not usually managed, and after logging "wolf" trees have been allowed to remain. As a result, such stands are extremely irregular in age, development and form. It is in this type of stand that American foresters wish to measure wood volumes as accurately as possible. This paper presents the results of an investigation to determine the possibility of applying the form-class method of volume determination to white oak¹⁶ as it is found in the second-growth hardwood stands of the eastern United States.

DEFINITION OF TERMS

Average of ratios. — For any group of trees the average of the ratios between the diameter at any given height and a base diameter at any arbitrary height. Algebraically, this is equivalent to

$$\frac{\frac{d_1}{D_1} + \frac{d_2}{D_2} + \frac{d_3}{D_3} + \frac{d_4}{D_4} + \dots + \frac{d_n}{D_n}}{n}$$

Ratio of average diameters. — For any group of trees the ratio between the average diameter at any given height and the average base diameter, chosen at an arbitrary height. Algebraically,

this is
$$\frac{\frac{d_1 + d_2 + d_3 + d_4 + \dots + d_n}{n}}{\frac{D_1 + D_2 + D_3 + D_4 + \dots + D_n}{n}}$$

Bark percentage. — The ratio between the double-bark thickness at any point on the tree stem and the diameter on bark at that point.

¹⁶ *Quercus alba* L.

Class mark. — The mean of the greatest and least variates that can occur in a given class.

Genuine form quotient. — The ratio between the tree diameter inside bark at four tenths of the distance from the tip to breast height and the corresponding diameter at eight tenths of the distance from the tip to breast height.

Genuine form class. — A range of genuine form quotients. In this paper the genuine form class takes the name of the class mark and each class is 0.05 wide. Consequently the 0.65 genuine form class includes all trees having a genuine form quotient ranging from 0.63 to 0.67 inclusive.

Riniker genuine form factor. — The ratio between the volume of a tree and the volume of a cylinder having a height equal to the height of the tree above breast height, and a diameter equal to the tree's diameter at eight tenths of the distance from the tip to breast height.

Standard deviation. — The square root of the mean of the squared deviations from the arithmetic mean. Algebraically, this is expressed as

$$\sigma = \sqrt{\frac{\sum d^2}{n}}.$$

Standard deviation of error of prediction. — The square root of the mean of the squared deviations from a line or plane of average relationship.

Standard deviation of a mean. — The standard deviation of the sample divided by the square root of the number of variates occurring in the sample. Algebraically, this is

$$\sigma_m = \frac{\sigma}{\sqrt{n}}.$$

SOURCE AND NATURE OF DATA

The data upon which the results of this study are based were obtained from the files of the Branch of Research of the United States Forest Service; they consist of separate cards for individual trees. Upon these cards are tabulated the stem diameters inside and outside bark at stated distances above the ground. Additional information, such as locality in which the

tree grew, age, crown class, clear length and width of crown, was given for some or all trees. From these data, measurements on 351 trees were found to be sufficiently complete for the purposes of this study. Most of these trees were from the following localities: southern Ohio, southwestern Connecticut, eastern West Virginia, New York and a few points in Maryland and Tennessee. Their distribution throughout the geographic range and by diameter and height classes is shown in Table I.

Plate XXVII shows an area of nearly pure white oak near Wardensville, West Virginia. This stand is practically even-aged; it followed a clear cutting made about 1775. A selective cutting was made about 1875, at which time species other than white oak were removed. Since that time the white oak alone has occupied the area.

The stand shown in Plate XXVIII is largely chestnut oak (*Quercus montana* Willd.) and black oak (*Quercus velutina* Lam.), also in the vicinity of Wardensville, West Virginia. In this type of stand white oak is commonly found.

METHOD OF TREATMENT

The diameter measurements at varying heights on the sample trees were made to the nearest tenth of an inch; in order to reduce these points of measurement to a comparable basis, the diameters inside and outside bark were plotted over height, one graph being made for each tree. Smooth curves were made to pass through each of the plotted diameters, one curve for the diameters inside bark, and one for diameters outside bark. From these curves were read diameters inside and outside bark for each tree at each tenth of the height of the stem above breast height.¹⁷ The double-bark thickness at each measuring point was obtained by subtracting the diameter inside bark from the diameter outside bark.

Diameter ratios were calculated for each diameter inside bark by dividing the diameter at any tenth of the distance from the

¹⁷ Through the remainder of this paper, unless a specific statement to the contrary is made, only the portion of the stem lying above the breast-high point is considered.

PLATE XXVII



Photograph by U. S. Forest Service

White oak in practically pure stand near Wardensville, West Virginia

PLATE XXVIII



Photograph by U. S. Forest Service
Mixed stand of black oak and chestnut oak near Wardensville, West Virginia

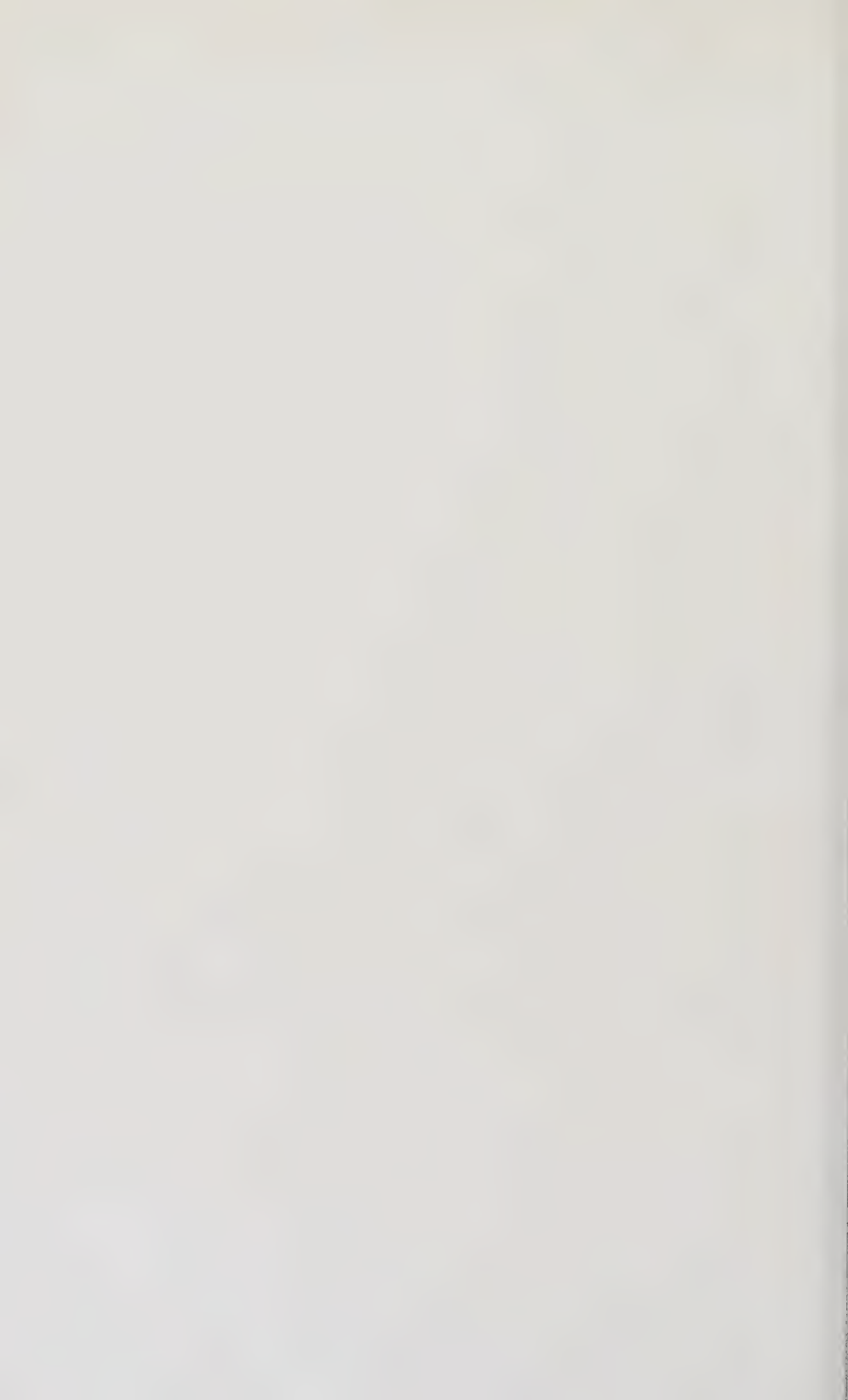


TABLE I
DISTRIBUTION OF TREES BY LOCALITY, AND BY DIAMETER AND HEIGHT CLASSES

Diameter at breast height in inches	GEOGRAPHIC LOCATION																			
	Connecticut				New York				West Virginia				Ohio				Other locations			
	Height classes *																			
	Number of trees																			
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Total
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* The twenty-foot height class contains trees whose heights are 19.6 to 29.5 feet inclusive. Other height classes are similarly limited.

tip by a base diameter taken at eight tenths from the tip. In this form diameters of trees of any size can be compared.

The computational labor of this study was materially lessened by the use of the Hollerith sorting and tabulating machines. Three cards were made for each tree. The first eighteen columns were identically punched for each of the three cards of each tree. The headings used for purposes of classification were: tree number, genuine form class, diameter at breast height in inches, total height of tree in feet, crown class, breaking point — that is, the point at which occurred a sharp break in the stem curve — geographical location, age, and the ratio between crown length and total height of tree. The last twenty-seven columns were punched differently for each of the three cards of each tree. On the first card in these columns were punched the actual diameters in inches at each tenth of the height of the stem. On the second card in these columns were punched diameter ratios at each tenth of the height of the stem. On the third card were punched double-bark thicknesses in inches at each tenth of the height of the stem.

In the analysis of the data the cards were sorted on the basis of the several variable factors, and totals of bark thickness, actual diameters and diameter ratios were easily obtained from the Hollerith tabulating machine.

In the earlier studies of tree form no satisfactory method of measuring root swell had been discovered, and consequently its influence upon the breast-high diameter was not determinable. Since the effect of a swollen base diameter is carried to the ratios at all the tenths of height, it is very important that the effect of root swell be eliminated. Behre (1, p. 678) has done this graphically by prolonging the main curve of the upper stem downward to the breast-high point. Hawley and Wheaton (6, p. 9) have also used this method in their work.

Hedeby (7, p. 202) computed the diameter free from root swell, i.e. the "normal" diameter breast-high for each tree, by passing a parabola through the ratios that represented the diameters at five tenths and nine tenths of the tree's length from the tip to breast height. In doing this he made the assumption that

root swell in oak does not extend up the stem as far as a point one tenth above breast height, and also that a parabolic curve will fit the normal curve of the stem in this portion of the trunk.

The stem curve of oak is not typically as regular as that of the western yellow pine (*Pinus ponderosa* Laws.) with which Behre (1, p. 678) worked, and the white oak stem curves shown in Figures 7a-c illustrate the difficulty of prolonging the curve for the upper portion of the stem downward to obtain graphically the normal breast-high diameter of any tree. Langsaeter (15, p. 129) quotes Heibel as having used the diameter nine tenths from the tip¹⁸ as the base for his ratio system. Langsaeter himself (15, p. 129) used as the basic diameter for his ratios the diameter at eight tenths from the tip. Realizing that a base plane lying above the zone of root swell has very real advantages over approximate methods of determining normal diameters, I have chosen the diameter at eight tenths from the tip as the base diameter for the system of ratios used in this study.¹⁹ Since the two diameters that comprise the form-class determining ratio should not lie too close together, the upper diameter of this ratio was chosen at four tenths from the tip.

This ratio was used in grouping the test trees into genuine form classes. For each genuine form class the average of the ratios at each tenth of the stem length was determined by adding the ratios and dividing by the number of trees in the class. This procedure may be open to objection on the ground that the average of a set of diameter ratios at a given height does not equal the ratio of the average diameter at that height to the average base diameter. Expressed algebraically, this is

$$\underbrace{\frac{d_1}{D_1} + \frac{d_2}{D_2} + \frac{d_3}{D_3} + \dots + \frac{d_n}{D_n}}_A \neq \underbrace{\frac{\frac{d_1 + d_2 + d_3 + \dots + d_n}{n}}{\frac{D_1 + D_2 + D_3 + \dots + D_n}{n}}}_B \quad (13)$$

¹⁸ Here the portion of the tree above the stump is considered, rather than the portion above breast height.

¹⁹ Figure 9, p. 313, shows that for white oak the diameter at eight tenths of the tree's length from the tip is free from the effects of root swell.

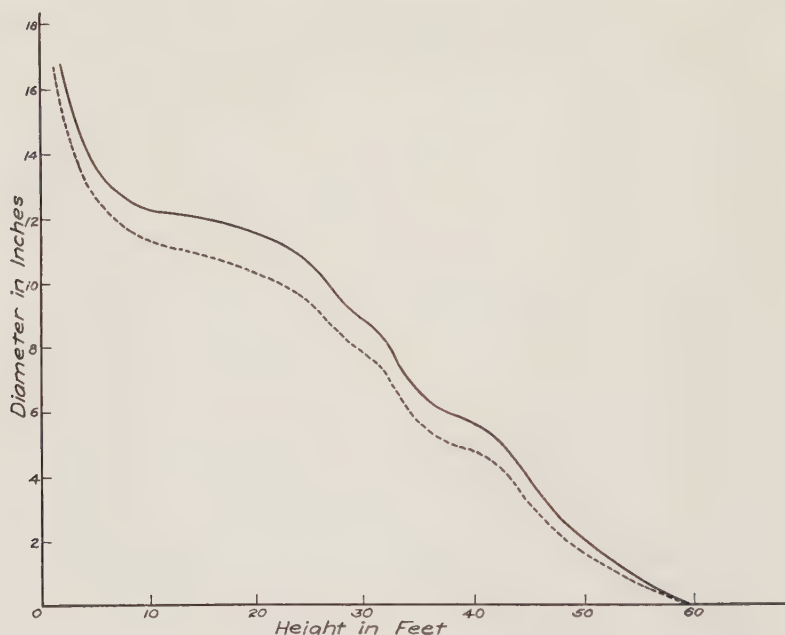


FIG. 7a. Stem curves of white oak: - - - inside bark; — outside bark

In any application of the results of such averaging to reconvert ratios to diameters the amount of inequality of these two expressions may be of importance.

The process of reconvertng ratios to diameters is sometimes employed to determine the diameter at a given height on trees of given diameter, form and total height. Wright uses this procedure (31, p. 113) as follows: "Read off on the plotted percentage taper curve the percentage diameters at these heights. Convert these percentage diameters to inches by multiplying diameter at breast height inside bark by the corresponding percentage diameter." For example, it may be desired to know the diameter at each tenth of the stem length for an average tree of a given species in the eight-inch diameter class, the fifty-foot height class and form class fifty. It is assumed that previous investigation has determined that the taper of the species

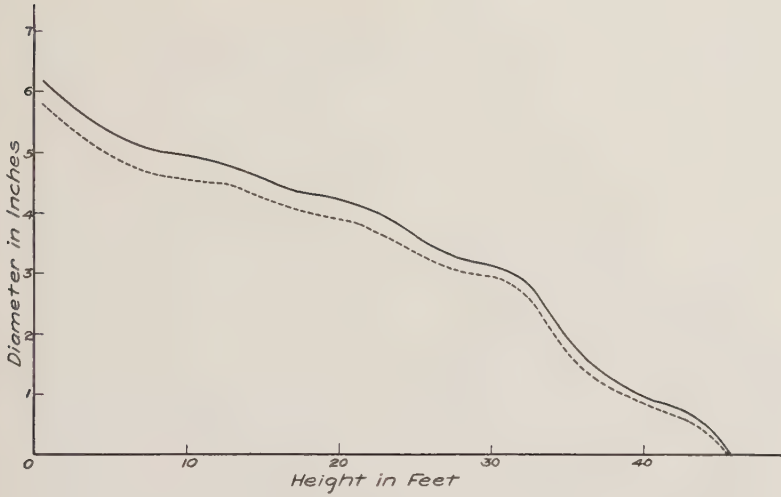


FIG. 7b. Stem curves of white oak: --- inside bark; — outside bark

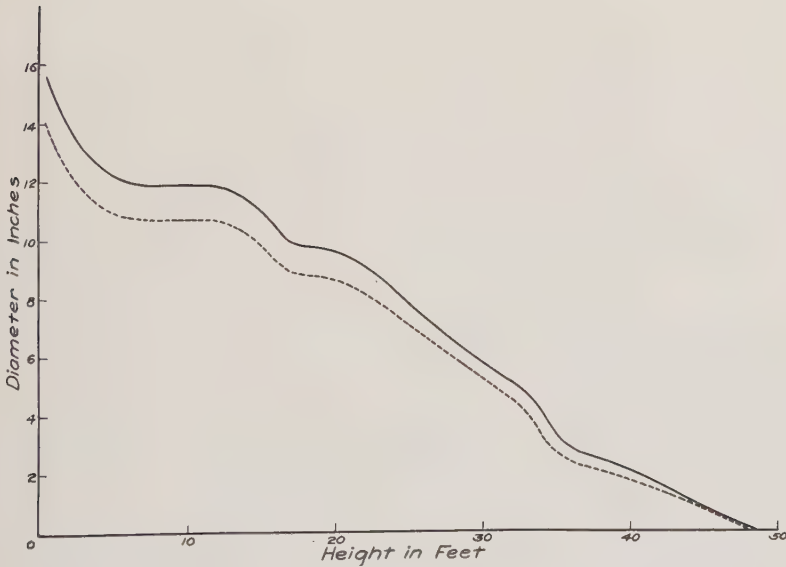


FIG. 7c. Stem curves of white oak: --- inside bark; — outside bark

conforms to the particular type of curve used. From the curve for form class fifty is read the average diameter ratio at the several heights, and this diameter ratio is converted to diameters by multiplying each ratio by the diameter at breast height, which is the base of the system of ratios used by Wright.

If trees in general conformed absolutely to any given stem curve, accurate volume determinations would be much simplified. The standard practice is to show that any species conforms relatively well to a certain stem curve by listing for each tenth of the stem's length the deviations of the average diameter ratios from the calculated ratio as given by the equation. If the deviations are small, the curve is said to express the stem taper well. An examination into the method of obtaining the average ratio for any tenth of the stem's length will show that certain fundamental errors may exist in the deviations of the averages of observed ratios from the ratios read from curves.

To show how these errors may influence the average diameter ratio at any given tenth of height, ten ratios at seven tenths of the tree's length from the tip were selected from the white oak trees falling in genuine form class 0.70. The ten ratios are as follows: 0.98, 0.88, 0.94, 0.98, 0.89, 0.91, 0.97, 0.88, 0.88 and 0.90, with the average at 0.921. This average is the result of averaging according to A of statement (13) on page 291. Within the range of diameters found in the white oak data, hypothetical diameters were assumed such that each pair of diameters would satisfy each of the foregoing ratios. An expression follows which gives the ratio of the average diameters. The first pair of diameters, one in the numerator and one in the denominator, when divided, gives the first ratio in the preceding list. The same thing is true for the remaining pairs of diameters:

$$\frac{6.0 + 13.1 + 5.8 + 4.9 + 12.8 + 5.1 + 6.4 + 10.1 + 11.1 + 8.1}{10} \div \frac{6.1 + 14.9 + 6.2 + 5.0 + 14.4 + 5.6 + 6.6 + 11.5 + 12.6 + 9.0}{10} = \frac{8.34}{9.19} = 0.908.$$

It is seen that the average ratio determined in this manner conforms to the procedure given in B of statement (13) on page 291. The average diameter at seven tenths of the distance from the

tip is 8.34 inches, and the average diameter at the base is 9.19 inches. The average ratios by the two procedures are seen to differ by 1.3 per cent. The ratio that was determined by the A process is the one that has been used in the past for comparisons to test the conformity to theoretical curves. In any event, the product of the ratio at the given height times the base diameter should give the average diameter at the given height. Accordingly, $0.921 \times 9.19 = 8.46$ instead of 8.34, which is the actual average diameter.

The main function of any form curve is to make possible the determination of the volume of a solid of the given form, or the determination of a diameter at any distance from the tip. In either case an approximation to the diameter at any point on the curve enter into consideration. So far as the ratio read from the curve fails to picture the average ratio that will correctly reproduce the average diameter, to that extent is the curve in error. If the average that is used as a norm by which the adequacy of the curved value is determined will not itself reproduce the average diameter at any given point when the basal diameter is known, then no assurance regarding the adequacy of the curve values is possible.

It must not be thought that a great error is usually made by this procedure in determining the adequacy of curve values. Especially is this true in the case of a species that has a regular taper. When, however, the stem taper is extremely variable from tree to tree, care should be taken to investigate the error that is introduced by using the average of a set of ratios instead of the ratio of averages. In studying the influence of various factors on tree form, some investigators have concerned themselves about the possibility of this or that small deviation of the average of ratios being significant. It is entirely possible that these small deviations may be the result of the difference between the average of the ratios and the ratio of the average diameters. Therefore, the curve value might easily be absolutely correct, or the error might be in the opposite direction, and the deviation be twice the amount that their results indicate.

It is easily seen that the amount of computational labor is materially lessened when the ratio of the average diameters

is used. At a specific height, the sum of the diameters of all trees falling in a given form class may be divided by the sum of all the diameters at breast height. In this manner the ten average ratios for a given form class may be calculated by only ten divisions. By the method of the average of ratios ten divisions must be made to determine the ratios for each tree. Though this may easily be done on the slide rule, the chances of error and the additional effort are certainly worth consideration.

In order to determine the variation in the average ratios for white oak, that resulted from the use of the two methods of procedure, the ratios at each tenth of height for each tree in the data were averaged by genuine form classes. Also, the upper diameters at each tenth of height were averaged by genuine form classes. For each genuine form class the average diameter at each tenth of the stem's height was divided by the average diameter at eight tenths from the tip, and a series of ratios of averages was thus constructed. The results are presented in Table II.

From a comparison of the A and B series of Table II it is apparent that very real differences between the series do exist. Genuine form classes 0.60, 0.65 and 0.70 show pronounced variations. In these genuine form classes the variations are not compensating, nor does either method of computation give consistently higher results. It must be pointed out that in certain instances variations of 1 per cent between the A and B series may be due to the fact that in one series the ratio may have been slightly over an even multiple of five-thousandths, and in the other series somewhat under an even multiple of five-thousandths. Consequently, in rounding off the ratio to two significant figures, the apparent deviation is greater than the actual deviation. This is not true, however, in all instances in which variation occurred. The magnitude of the variation depends largely upon the distribution of the variates about the mean for any point of measurement on the stem. This distribution will vary with any set of data that may be collected.

It will be seen from Table II that, for the most part, ratios are given to two significant figures. Since tree diameters are

TABLE II
COMPARISON BETWEEN THE AVERAGE OF RATIOS AND THE
RATIO OF AVERAGE DIAMETERS

Genuine form class	Series	Percentage of distance from tip to breast height										Basis no.of trees
		100	90	80	70	60	50	40	30	20	10	
		Average ratios inside bark										
.40	A*	1.20	1.07	1.00	.91	.79	.60	.40	.28	.19	.09	12
.40	B	1.20	1.06	1.00	.92	.80	.59	.41	.28	.18	.08	12
.45	A	1.16	1.05	1.00	.92	.79	.62	.45	.33	.22	.11	13
.45	B	1.16	1.05	1.00	.92	.79	.63	.46	.33	.21	.11	13
.50	A	1.18	1.07	1.00	.93	.83	.67	.50	.38	.26	.13	32
.50	B	1.19	1.06	1.00	.94	.83	.68	.51	.38	.25	.13	32
.55	A	1.19	1.07	1.00	.93	.83	.69	.55	.40	.27	.13	74
.55	B	1.19	1.07	1.00	.93	.83	.69	.56	.40	.26	.12	74
.60	A	1.20	1.08	1.00	.93	.85	.75	.60	.45	.30	.16	71
.60	B	1.22	1.08	1.00	.94	.86	.75	.62	.46	.31	.15	71
.65	A	1.20	1.08	1.00	.93	.85	.76	.65	.49	.33	.17	70
.65	B	1.18	1.07	1.00	.92	.85	.75	.64	.48	.31	.16	70
.70	A	1.19	1.08	1.00	.93	.87	.80	.70	.53	.35	.18	54
.70	B	1.18	1.06	1.00	.93	.87	.79	.69	.52	.34	.18	54
.75	A	1.22	1.08	1.00	.94	.88	.83	.74	.55	.36	.17	18
.75	B	1.22	1.08	1.00	.94	.87	.82	.74	.55	.35	.17	18
.80	A	1.25	1.08	1.00	.96	.88	.85	.79	.62	.38	.18	6
.80	B	1.24	1.06	1.00	.96	.89	.85	.80	.63	.38	.18	6
.85	A and B	1.26	1.09	1.00	.95	.92	.87	.83	.61	.38	.17	1

* A = average of ratios; B = ratio of average diameters.

read to tenths of inches, and most of the upper diameters were less than ten inches, two significant figures in the diameters of the higher portion of the stem are all that the data justify.

four tenths and eight tenths of the distance from the tip is obtained by grouping the data by this method and calculating the standard deviation from the mean of the diameter ratios at each tenth of the stem's height. The standard deviations at various distances from the tip are given in Table III.

From these data it is apparent that the variation resulting from a deviation of one standard unit from the mean ratio for any form class at any given height is relatively large. For purposes of comparison the following table of standard deviations of white pine (*Pinus Strobus* L.) is taken from the work of Gevorkiantz and Hosley (5, p. 35). Although the authors do not definitely define the significance of the tabulated values, it is inferred from the context that they represent the standard deviations of the diameter ratios at each tenth of the stem's height about the mean ratio for each form class.

TABLE IV

STANDARD DEVIATIONS OF WHITE PINE DIAMETER RATIOS

Distance from tip to breast height, as a percentage	Standard deviation from mean of form class. Aver- age for all form classes
.90	± 2.8
.80	± 2.9
.70	± 2.9
.60	± 2.5
.50	
.40	± 2.2
.30	± 3.2
.20	± 3.3
.10	± 3.0

In order to compare the standard deviations in Table IV with those in Table III, divide the standard deviations in Table IV by 100. This comparison shows white oak to have greater variability of stem form than does the white pine with which Gevorkiantz and Hosley worked.

A comparison of the curves of numbers of white oak trees shows that many of them have a point at which there is a decided break in the trend of the curve. The possibility of classifying trees into form classes on the basis of the percentage of

height above breast height at which this break occurs has been investigated. The deviations of the ratios from the average ratio at the various heights are shown in Table V.

TABLE V
STANDARD DEVIATION OF ERRORS FROM MEAN RATIO AT
GIVEN TENTHS OF DISTANCE FROM TIP TO BREAST HEIGHT
Breaking-point Classification

Breaking-point class tenths of height	Percentage of distance from tip to breast height			Basis no. of trees
	100	60	20	
	Standard deviation of errors *			
Z	.077	.050	.060	144
3	.056	.073	.055	24
4	.066	.040	.067	51
5	.062	.042	.070	63
6	.073	.044	.057	50
7	.044	.035	.093	16
				348

* In order to shorten a very long computation, the standard deviations at 100, 60 and 20 per cent from the tip were the only ones calculated.

Class Z includes those trees whose curves showed no definite breaking point. Classes 3, 4, . . . , contain trees whose curves had a definite break at 3, 4, . . . , tenths of the height above breast height. One tree fell in Class 2 and two trees fell in Class 8. These were not included in Table V, since the standard deviations would have little significance.

Comparison of Tables III and V shows no superiority of the breaking-point system of classification over the genuine form-class method. Had the standard deviations at four tenths from the tip been compared, the genuine form-class method would have been superior to the breaking-point classification. The basis upon which grouping into genuine form classes is done makes this result inevitable. By the genuine form-class system, trees

are grouped into classes on the basis of the diameter ratio at four tenths from the tip, and the maximum dispersion of ratios about their mean at this height is 0.02. This will give a small standard deviation at this point, whereas the breaking-point classification may be expected to have a much greater dispersion about the mean at four tenths from the tip. It is apparent from Tables III and V that by neither method of classification can white oak be grouped into classes that show consistent form.

STEM FORM OF WHITE OAK

From the preceding paragraph the genuine form-class grouping is seen to be preferable to the breaking-point system of classification. Is there any significant trend in the form variation from one genuine form class to another? Figure 8 presents graphically the B-values from Table II. From this figure it is apparent that

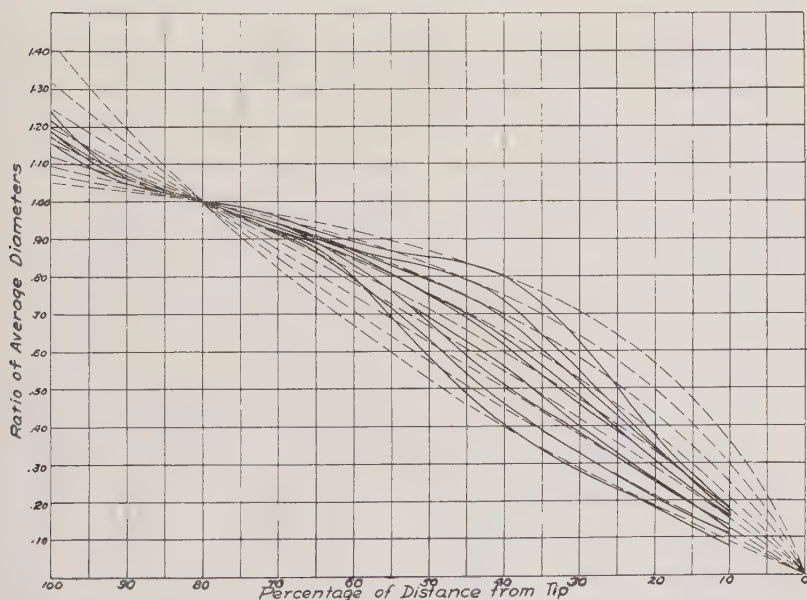


FIG. 8. Comparison of empirical curves of white oak and theoretical curves of Behre's formula: --- curves of Behre's formula; — white oak curves

TABLE VI

RINIKER GENUINE FORM FACTORS FOR DIFFERENT FORM CLASSES

Genuine form class	Series A	Series B	Basis no. of trees
.40	.50	.49	12
.45	.51	.51	13
.50	.53	.53	32
.55	.54	.54	74
.60	.57	.57	71
.65	.58	.57	70
.70	.60	.59	54
.75	.62	.61	18
.80	.65	.65	6
.85	.65	.65	1
			351

below seven tenths from the tip there is a striking agreement among all genuine form classes. Above this point the form-class curves are well defined. Since the curves vary only in the upper portion, is the variation sufficient to warrant the use of a system of form classification? To determine this, the average diameter ratios for the whole mass of data were found for eight, nine and ten tenths from the tip. These averages were used for all genuine form classes below eight tenths from the tip. By this procedure the A and B values in columns headed 100 and 90 in Table II are as follows for all genuine form classes.

	100	90
A	1.20	1.08
B	1.20	1.07

With these values for the lower two tenths of the stem's height and the values from Table II at the other heights, Riniker's genuine form factors were computed by Simpson's rule ²⁰ for each form class for both the A and B series.

²⁰ Simpson's rule is expressed as follows:

$$f = \frac{\Delta x}{3} [y_0^2 + y_{10}^2 + 4(y_1^2 + y_3^2 + \dots + y_9^2) + 2(y_2^2 + y_4^2 + \dots + y_8^2)]$$

where y is expressed as a diameter ratio, taken from Table II, and f is the Riniker genuine form factor.

TABLE VII

DEVIATION * OF RATIOS OF AVERAGE DIAMETERS FROM DIAMETER
RATIOS READ FROM BEHRE'S CURVE

Genuine form class	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Difference between ratios of average diameters inside bark and diameter ratios read from Behre's curve										
.40	-.23	-.14	00	+.10	+.13	+.06	+.01	-.01	00	-.01	12
.45	-.16	-.11	00	+.07	+.08	+.05	+.01	00	00	+.01	13
.50	-.06	-.06	00	+.06	+.08	+.06	+.01	00	00	+.01	32
.55	-.01	-.03	00	+.03	+.04	+.02	+.01	-.02	-.03	-.03	74
.60	+.07	00	00	+.03	+.04	+.04	+.02	-.01	-.02	-.03	71
.65	+.06	+.01	00	-.01	00	-.01	-.01	-.05	-.07	-.05	70
.70	+.09	+.01	00	-.01	-.01	-.01	-.01	-.06	-.10	-.07	54
.75	+.15	+.04	00	-.02	-.03	-.01	-.01	-.09	-.15	-.13	18
.80	+.19	+.03	00	-.01	-.03	-.02	00	-.08	-.19	-.18	6
											350

* Curve is taken as normal; deviations are plus or minus from the curve.

In the Riniker genuine form factor the variation between certain intermediate genuine form classes is very slight, although the form deviation from the lowest to highest genuine form class is a considerable one.

An attempt has been made to determine whether or not a set of curves, such as Behre's, can reproduce in any satisfactory manner the data actually measured. Table VII shows for each genuine form class the difference between the measurements on white oak and the diameter ratios from Behre's curve.

The diameter ratios from Behre's curve for genuine form classes 0.55, 0.60 and 0.65 fit the observed data rather well. In the lower portions of the stem the lower genuine form classes have a negative deviation and the higher genuine form classes a positive one. This fact, as well as the curves in Figure 8, points to the conclusion that all genuine form classes have a similar form in the lower stem portions. In the upper stem portions practically all deviations are negative, but in the lower genuine form classes the deviations are slight.

TABLE VIII
INFLUENCE OF HEIGHT UPON TAPER WITHIN GIVEN GENUINE FORM CLASS

Height class * in feet	Average diameter breast- high in inches	Percentage of distance from tip to breast height										Basis no. of trees
		100	90	80	70	60	50	40	30	20	10	
		Ratios of average diameters inside bark Genuine form class .55										
20	3.6	1.17	1.08	1.00	.92	.77	.66	.55	.38	.28	.17	2
30	4.3	1.20	1.06	1.00	.94	.82	.68	.54	.43	.31	.15	5
40	6.6	1.16	1.07	1.00	.92	.80	.67	.55	.42	.27	.14	16
50	9.2	1.20	1.07	1.00	.93	.84	.70	.55	.39	.26	.13	29
60	12.7	1.23	1.07	1.00	.94	.85	.72	.56	.40	.25	.12	11
70	13.3	1.21	1.08	1.00	.96	.84	.65	.56	.44	.29	.14	4
80	14.5	1.19	1.09	1.00	.93	.87	.74	.56	.40	.26	.13	7
												74
Genuine form class .60												
20	3.6	1.18	1.06	1.00	.92	.86	.76	.59	.41	.29	.15	3
30	3.8	1.23	1.12	1.00	.93	.87	.77	.60	.45	.31	.17	7
40	6.4	1.19	1.07	1.00	.93	.84	.73	.60	.46	.31	.16	11
50	8.6	1.20	1.08	1.00	.93	.85	.75	.60	.44	.29	.15	25
60	9.0	1.20	1.07	1.00	.93	.85	.76	.61	.47	.32	.17	11
70	13.8	1.21	1.06	1.00	.92	.85	.73	.60	.44	.30	.15	8
80	14.0	1.22	1.09	1.00	.94	.86	.75	.62	.47	.29	.13	6
												71

* The twenty-foot height class includes trees ranging in height from 19.6 feet to 29.5 feet. Other height classes follow similarly.

TABLE VIII (continued)
INFLUENCE OF HEIGHT UPON TAPER WITHIN GIVEN GENUINE FORM CLASS

Height class * in feet	Average diameter breast-high in inches	Percentage of distance from tip to breast height										Basis no. of trees
		100	90	80	70	60	50	40	30	20	10	
		Ratios of average diameters inside bark Genuine form class .65										
20	3.6	1.22	1.10	1.00	.92	.86	.78	.66	.50	.34	.20	2
30	3.6	1.20	1.07	1.00	.93	.84	.73	.64	.52	.37	.19	7
40	6.6	1.18	1.08	1.00	.92	.85	.77	.66	.49	.32	.16	16
50	7.8	1.20	1.07	1.00	.93	.85	.77	.65	.49	.32	.16	25
60	8.8	1.22	1.08	1.00	.93	.86	.76	.64	.48	.34	.18	11
70	11.9	1.17	1.07	1.00	.88	.85	.82	.65	.44	.28	.13	2
80	15.4	1.19	1.07	1.00	.94	.86	.78	.66	.47	.29	.14	5
90	14.0	1.19	1.10	1.00	.93	.87	.76	.65	.50	.30	.13	2
												70

* The twenty-foot height class includes trees ranging in height from 19.6 feet to 29.5 feet. Other height classes follow similarly.

It may be thought that the wide variations in Table VII are due principally to large individual variations arising from irregular branching in the tree crowns. A summary similar to Table VII has been prepared for those trees whose stem curve showed no definite breaking point. In this summary the deviations are consistently smaller than the deviations in Table VII, but the tendencies for variation are similar; the only exception is a greater number of positive deviations in the upper portions of stems of low genuine form class.

Figure 8 presents graphically Table VII. The diameter ratios from Behre's curves are shown in broken lines, one line for each genuine form class, whereas the ratios of average diameters for the white oak data are shown in solid lines. The numerical designation of the curve representing any form class can be determined by reading the ordinate of the curve at 40 per cent of the distance from the tip. At this abscissa Behre's curve for form class 0.50 has an ordinate of 0.50, etc. The empirical curves for the white oak data very nearly agree with the theoretical curves at this abscissa. It is plainly seen, however, that the curves of Behre's equation do not represent the stem form of white oak.

THE EFFECT OF CERTAIN FACTORS UPON TAPER

The influence of height upon form of white oak has been studied from two viewpoints. The influence of height upon trees falling within the same genuine form classes is seen from the results that are tabulated in Table VIII. For the purpose of this comparison three of the stronger genuine form classes are presented. Table VIII shows that no consistent tendency for variation is associated with height within any given genuine form class.

The influence of height on the ratios of average diameters at the various tenths of the stem's height has been studied independently of genuine form class, and the results are presented in Table IX. For this tabulation all the sample trees were divided into ten-foot height classes, and the ratios of average diameters at the several tenths were calculated for the height classes included within the data.

TABLE IX

RATIOS OF AVERAGE DIAMETERS INSIDE BARK FOR WHITE OAK
SORTED ACCORDING TO HEIGHT

Height class* in feet	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Ratios of average diameters inside bark										
20	1.18	1.07	1.00	.92	.83	.71	.56	.42	.31	.17	14
30	1.21	1.09	1.00	.92	.82	.71	.58	.46	.32	.17	30
40	1.18	1.07	1.00	.92	.82	.71	.59	.44	.29	.15	69
50	1.19	1.07	1.00	.93	.85	.73	.59	.43	.28	.14	125
60	1.21	1.07	1.00	.93	.85	.75	.62	.46	.31	.16	53
70	1.21	1.07	1.00	.93	.85	.71	.58	.43	.28	.14	19
80	1.20	1.08	1.00	.93	.86	.77	.64	.48	.30	.14	30
90	1.20	1.07	1.00	.95	.88	.80	.71	.52	.32	.15	11
											351

* The twenty-foot height class includes trees ranging in height from 19.6 feet to 29.5 feet. Other height classes follow similarly.

From this tabulation a slight tendency toward a fuller form in the larger height classes is apparent. More particularly is this true of the ratio of average diameters at three tenths to seven tenths from the tip.

The influence of the ratio between crown length and total height of tree upon the taper throughout the stem's length was investigated for white oak by separating those trees upon which crown-length measurements were available into ratio classes where the ratios fell into ten-unit classes varying between 0.196 and 0.295, etc. Table X presents the ratios between the average diameters at various heights for the different crown-ratio classes.

It appears from this tabulation that trees with long crowns have a slightly greater amount of taper in the upper portion of the stem than do trees with short crowns.

The influence of crown class on the taper of white oak trees is shown in Table XI.

TABLE X

RATIOS OF AVERAGE DIAMETERS FOR WHITE OAK, SORTED ACCORDING
TO RATIO OF CROWN LENGTH TO TOTAL LENGTH

Ratio be- tween crown length and total length	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Ratios of average diameters inside bark										
.20	1.14	1.03	1.00	.89	.77	.70	.59	.44	.30	.15	1
.30	1.18	1.07	1.00	.91	.83	.75	.63	.49	.34	.18	15
.40	1.17	1.06	1.00	.93	.86	.75	.60	.43	.28	.14	35
.50	1.20	1.07	1.00	.94	.85	.71	.55	.39	.25	.12	44
.60	1.18	1.06	1.00	.92	.83	.70	.55	.39	.25	.12	25
.70	1.18	1.09	1.00	.88	.72	.57	.45	.34	.21	.11	2
											122

TABLE XI

RATIOS OF AVERAGE DIAMETERS FOR WHITE OAK,
SORTED ACCORDING TO CROWN CLASSES

Crown class	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Ratios of average diameters inside bark										
Dominant . . .	1.17	1.06	1.00	.92	.83	.72	.57	.41	.26	.13	16
Codominant .	1.20	1.07	1.00	.93	.85	.72	.56	.39	.25	.12	37
Intermediate .	1.18	1.07	1.00	.93	.85	.73	.58	.43	.29	.14	62
Oppressed (Suppressed) .	1.13	1.03	1.00	.91	.75	.63	.52	.40	.27	.13	3
											118 *

* Trees measured in southwestern Connecticut.

The conclusion drawn from these data is that form variation is independent of crown class in white oak, although the sample in the oppressed class is too small to give more than an indication relative to the conditions in that class.

When the white oak data are sorted into age classes, and the ratio of the average diameters at the various heights computed and averaged, the results are as shown in Table XII. Age apparently has no influence whatsoever on the taper of white oak.

TABLE XII

RATIOS OF AVERAGE DIAMETERS FOR WHITE OAK,
SORTED ACCORDING TO AGE

Age * in years	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Ratios of average diameters inside bark										
20	1.17	1.07	1.00	.94	.84	.71	.57	.43	.30	.17	6
30	1.23	1.09	1.00	.92	.84	.73	.61	.46	.31	.16	41
40	1.20	1.08	1.00	.93	.85	.75	.62	.47	.32	.17	36
50	1.20	1.08	1.00	.94	.86	.75	.61	.45	.30	.15	56
60	1.18	1.07	1.00	.93	.84	.71	.57	.42	.28	.14	87
70	1.19	1.07	1.00	.93	.85	.75	.61	.45	.31	.15	45
80	1.19	1.07	1.00	.89	.80	.68	.52	.38	.25	.12	9
90	1.23	1.05	1.00	.90	.82	.72	.57	.41	.26	.12	5
130	1.21	1.05	1.00	.98	.89	.71	.55	.40	.27	.14	1
150	1.20	1.05	1.00	.95	.87	.72	.66	.48	.27	.12	1
160	1.23	1.05	1.00	.99	.93	.69	.60	.55	.36	.16	1
											288

* The twenty-year age class embraces trees whose ages vary from 20 to 29 years, inclusive.

Table XIII shows no significant relationship between diameter at breast height and the ratios of average diameters at various heights on the stem of the tree.

TABLE XIII

RATIOS OF AVERAGE DIAMETERS FOR WHITE OAK, SORTED
ACCORDING TO DIAMETER AT BREAST HEIGHT

Diameter breast- high in inches*	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Ratios of average diameters inside bark										
3	1.17	1.07	1.00	.92	.82	.71	.59	.47	.33	.18	22
4	1.21	1.09	1.00	.92	.82	.70	.55	.41	.29	.16	19
5	1.18	1.08	1.00	.91	.83	.72	.62	.47	.32	.17	22
6	1.19	1.10	1.00	.92	.82	.73	.61	.48	.32	.16	23
7	1.20	1.08	1.00	.93	.85	.75	.63	.48	.32	.17	46
8	1.19	1.07	1.00	.94	.86	.75	.62	.47	.31	.16	63
9	1.20	1.07	1.00	.94	.84	.73	.59	.44	.30	.15	40
10	1.19	1.07	1.00	.94	.86	.74	.59	.42	.27	.13	25
11	1.17	1.07	1.00	.92	.84	.72	.55	.39	.26	.13	24
12	1.17	1.06	1.00	.91	.82	.72	.59	.42	.28	.14	9
13	1.24	1.08	1.00	.92	.84	.73	.60	.43	.28	.14	14
14	1.21	1.06	1.00	.95	.86	.74	.60	.44	.26	.13	13
15	1.19	1.07	1.00	.93	.87	.78	.66	.53	.31	.13	9
16	1.21	1.07	1.00	.92	.85	.74	.62	.46	.27	.12	10
17	1.21	1.07	1.00	.96	.87	.79	.65	.44	.29	.15	7
18	1.19	1.08	1.00	.93	.90	.77	.66	.54	.33	.13	3
19	1.23	1.06	1.00	.94	.87	.73	.58	.42	.29	.15	2
											351

* The three-inch diameter class embraces trees whose diameters vary from 2.6 to 3.5 inches, inclusive.

From the preceding tabulations it appears that the taper of white oak is not correlated with crown class, age or diameter at breast height, and is but slightly influenced by height and crown length.

The influence of distribution upon form is an important consideration for the forester, since the tables of wood volume for any species depend upon the similarity of form of trees grown in the several parts of the commercial range of the species. Table XIV indicates that the form of white oak in southeastern Ohio

is very similar to that grown in eastern West Virginia. The ratios of average diameters of the New York trees are markedly lower at 30, 40 and 50 per cent of the distance from the tip than are the West Virginia and Ohio trees. Connecticut trees conform very well to the Ohio and West Virginia trees for the lower half of the stem, but are considerably smaller in the upper half. Since variations in the upper portion of the stem affect the volume a relatively smaller amount than equivalent variations in the lower portion, the form factor is affected no more than is shown in column 2 of Table XIV.

TABLE XIV

RATIOS OF AVERAGE DIAMETERS FOR WHITE OAK, SORTED
ACCORDING TO LOCALITY

Locality	Riniker genuine form factor	Percentage of distance from tip to breast height										Basis no. of trees
		100	90	80	70	60	50	40	30	20	10	
		Ratios of average diameters inside bark										
Connecticut...	.55	1.19	1.07	1.00	.93	.84	.72	.57	.41	.27	.13	122
New York....	.56	1.20	1.09	1.00	.92	.82	.71	.58	.45	.31	.17	72
West Virginia..	.58	1.20	1.07	1.00	.94	.86	.76	.64	.48	.30	.15	55
Ohio.....	.58	1.21	1.08	1.00	.93	.86	.75	.63	.48	.32	.16	95
												344

ROOT SWELL

Since the common practices for eliminating root swell²¹ cannot be satisfactorily applied to the irregularly tapering stem of white oak, a conception of the amount of root swell present has been determined by diameter classes. Average diameters inside bark at each tenth of the stem's height were determined for all trees falling in each diameter class. A series of curves was made in which height was plotted on the horizontal and average diameter inside bark on the vertical scales. Since most of the diameter classes contained a sufficient number of trees to

²¹ See page 290.

give significant averages,²² a set of relatively smooth curves resulted. In all cases root swell was apparent. To determine the amount of this distortion, the smooth upper portion of the curve was prolonged downward, and the amount of swelling read from the graph. From Figure 9 the amounts of root swell at breast height were read, and are presented in Table XV.

TABLE XV
AMOUNT OF ROOT SWELL AT BREAST HEIGHT

Diameter class *	Root swell	Basis
in	in	no. of
inches	inches	trees
3	0.25	22
4	0.35	19
5	0.20	22
6	0.30	23
7	0.45	46
8	0.60	63
9	0.70	40
10	0.85	25
11	0.30	24
12	0.37	9
13	1.05	14
14	1.30	13
15	1.10	9
16	1.20	10
17	1.42	7
		346

* The four-inch diameter class embraces trees whose diameters at breast height outside bark vary from 3.6 inches to 4.5 inches, inclusive.

Figure 9 indicates that an appreciable amount of root swell may frequently be expected to reach a height of one tenth of the stem's length above breast height, although it is doubtful whether the swell reaches as high as two tenths in any considerable number of trees.

²² In Figure 9 the diameter-height curves for the 16- and 17-inch trees have not been continued to the tip. Insufficiency of data in these diameter classes caused irregularities in the upper portions. However, the smooth lower portion gives a satisfactory conception of the amount of root swell.

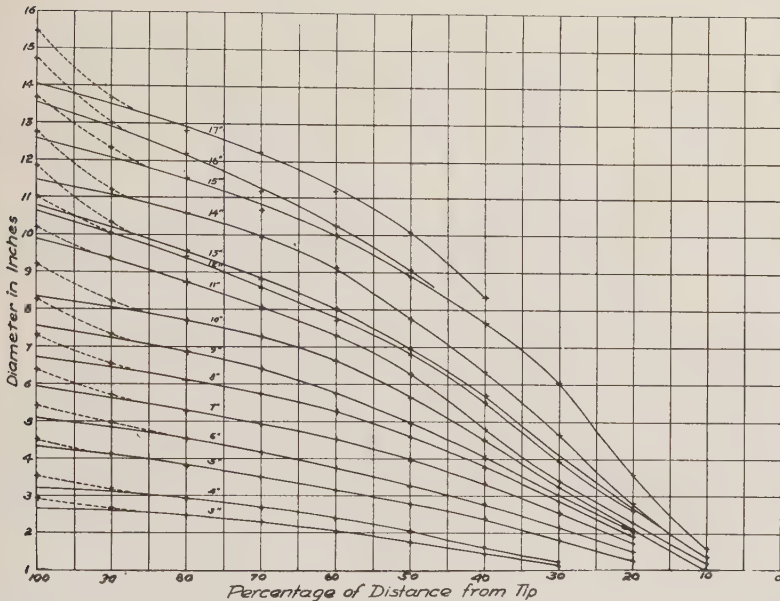


FIG. 9. Root swell as shown by diameter-height curves. --- departure of the actual diameter curve from the normal curve of the upper stem when it is prolonged downward to breast height

BARK DEVELOPMENT

Investigations into bark thickness have been carried on by several investigators on several species, with varying results. Mattsson (16, p. 883) finds that in European larch (*Larix europaea* DC.) the bark thickness expressed as a percentage of the diameter of the wood is a minimum in the middle portion of the tree stem. At this point the bark forms 10–15 per cent of the diameter of the wood. For the lower stem portion the bark percentage increases to 15–20 per cent, and Mattsson states that toward the tip the bark percentage approaches infinity.²³

²³ Since there is a ratio at the tip determined by the differentiation of embryonic tissue at the growing tip, it is obvious that this statement is not literally true. It would be clarified by changing "infinity" to "that ratio initially determined by primary differentiation."

Jonson (10, p. 305), working with Scotch pine, finds that the bark in percentage of diameter inside bark at the various heights on the tree stem is thinnest near the middle of the stem's height. Toward either end this percentage increases, but is greater at breast height than it is at one tenth the distance from the tip to breast height.

Pemberton's investigations (20, p. 46) on redwood (*Sequoia sempervirens* Endl.) indicate that the ratio between bark thickness and diameter²⁴ decreases from the base to the tip of the tree. He also concludes that the bark-diameter ratio is practically constant at breast height for any diameter of trees.

Hedeby (7, p. 212) shows graphically that the bark percentage is greatest near the tip of Swedish oak. Lower on the stem the bark percentage gradually decreases, and is a minimum at the base of the tree. His researches also show that the bark percentage at breast height is greatest for the lower diameter classes, and diminishes gradually as the diameter at breast height increases.

Investigations of Gevorkiantz and Hosley (5, p. 38) on white pine bark in central New England indicate that the relationship between bark and total diameter is about the same at breast height as it is at one tenth the distance from the tip to breast height. From either end the bark percentage gradually approaches a minimum at the center of the tree.

The white oak data, as presented in Table XVI, confirm the findings of Hedeby (7, p. 212).

It is seen that the bark percentage at breast height diminishes more or less regularly from the lower to the higher diameter classes. Since the greater part of the irregularity is found in the higher diameter classes where the data are weakest, at least a part of the irregularity may be attributed to this weakness. Further irregularity is possibly due to the distribution of the size classes throughout the several localities in which the data were gathered. It is seen from Table I (p. 289) that several of the diameter classes are represented by sample trees from a

²⁴ In the absence of a direct statement, the inference is that this diameter is diameter on bark.

TABLE XVI

DOUBLE-BARK THICKNESS DIVIDED BY DIAMETER ON BARK
AT GIVEN TENTHS FROM TIP TO BREAST HEIGHT

Diameter breast high * in inches	Percentage of distance from tip to breast height										Basis no. of trees
	100	90	80	70	60	50	40	30	20	10	
	Double-bark thickness divided by diameter on bark										
3	.11	.12	.12	.12	.12	.13	.13	.13	.14	.15	22
4	.11	.12	.12	.12	.12	.11	.13	.14	.15	.17	19
5	.11	.11	.12	.12	.12	.13	.14	.15	.17	.19	22
6	.11	.10	.11	.11	.12	.12	.12	.14	.15	.17	23
7	.099	.096	.098	.10	.10	.11	.12	.13	.15	.17	46
8	.092	.096	.10	.11	.11	.12	.12	.14	.15	.17	63
9	.091	.097	.10	.11	.11	.12	.13	.14	.15	.17	40
10	.090	.093	.099	.10	.11	.12	.12	.14	.15	.18	25
11	.084	.088	.091	.098	.10	.11	.12	.14	.16	.17	24
12	.087	.088	.094	.10	.11	.12	.13	.14	.15	.16	9
13	.084	.092	.091	.092	.095	.099	.10	.13	.14	.16	14
14	.089	.093	.092	.091	.092	.10	.11	.12	.15	.16	13
15	.087	.087	.088	.084	.083	.083	.085	.090	.12	.15	9
16	.080	.086	.085	.084	.085	.091	.10	.11	.14	.15	10
17	.088	.10	.10	.098	.094	.095	.10	.12	.15	.17	7
											346

* The four-inch diameter class embraces trees whose diameters vary from 3.6 inches to 4.5 inches, inclusive.

single locality. It may be that trees of the same diameter at breast height will exhibit different bark thicknesses in different parts of the range of the species. In order to determine whether this supposition was tenable, trees from the four localities that furnished the greater part of the data were sorted on diameter at breast height, and the bark percentage at breast height for diameter classes was calculated by localities. No diameter class was represented in all the localities, but where diameter classes were found in more than one locality there was not universal agreement in bark percentage. In three diameter classes there

were significant averages obtainable in two localities, and in these there was close agreement in bark percentage. When the averages in one locality were based on a small number of trees, disagreement in bark percentage was common. It is impossible to say definitely whether the variation is due to inadequate data or to regional variation, although it is the author's personal feeling that the latter possibility is of considerable importance.

This belief is strengthened by the manner in which the actual bark thickness in inches varies from one diameter class to another. In Table XVII the double-bark thickness was calculated by multiplying the bark percentage at breast height, taken from Table XVI, by the actual average diameter at breast height for each diameter class.

TABLE XVII

BARK THICKNESS IN INCHES FOR GIVEN DIAMETER CLASSES

Diameter class in inches	Bark thickness in inches	Basis no. of trees
3	0.36	22
4	0.44	19
5	0.56	22
6	0.67	23
7	0.70	46
8	0.74	63
9	0.83	40
10	0.91	25
11	0.93	24
12	1.04	9
13	1.08	14
14	1.25	13
15	1.31	9
16	1.28	10
17	1.49	7
		346

It is observed that the break in the regular trend of the bark increase at the seven-inch diameter class coincides with the increasing influence of the Ohio trees and the diminishing influence of the New York trees.²⁵ Irregularity in the higher diameter

²⁵ See Table I, page 289.

classes cannot be attributed with definiteness to this cause, since in no locality are strong averages found.

Within any diameter class the bark percentage increases from breast height to the tip, and the bark percentage at the tip appears to be approximately constant for all diameter classes. In the greater number of diameter classes the rate of increase in bark percentage is relatively slow in the lower part of the stem, and rises sharply at about four tenths of the distance from the tip to breast height.

This diminishing in bark percentage from the tip to the base of white oak trees is very easily accounted for. Near the tip of the stem all the bark elements formed by the cambium and cork cambium are present. Casual observation on a few white oak sections having two or three annual rings shows that at this age the thickness of bark is about one fifth to one third of the radius of xylem tissue. As the bark ages it forms into ridges that are susceptible to scaling off by animals and the bending action of the wind. Natural weathering also disintegrates the exposed bark portions. Consequently, not all the bark formed by the tree is present in the lower portions of the stem, and the radius of the xylem cylinder does not diminish. This causes a reduction in the bark percentage at any given point on the stem as the age of the stem at that point increases.

Büsgen and Münch (2, p. 144) explain the variation in bark percentage in pine²⁶ in the following manner: "The raggedness of the surface of bark is caused by the stretching of the rind associated with the growth in thickness of the stem of the tree. The cork cambium and the other living constituents of the rind follow it by cell multiplication, cell growth and the formation of larger intercellular spaces, so that these parts of the rind behave like a coat which enlarges of its own accord with the growth of the wearer. The dead bark cannot do this and therefore acquires fissures whose direction generally follows the length of the stem and depends in individual cases on the constitution of the bark. . . . In the pine the shedding of the scales in the upper part of the stem occurs early and in thin layers, while further down

²⁶ Presumably Büsgen and Münch here refer to Scotch pine.

the scales remain on for a long time. Above, therefore, there are always present new, brightly coloured scales, whilst below every scale has time to change colour under the influence of the weather and the impurities that settle upon it."

In conclusion, it appears that bark percentage at given stem heights varies from species to species according to the peculiarities of bark structure that are characteristic of the different species.

FORM CLASS DETERMINATION

Many investigators have expressed themselves concerning the best method for the determination of form class. Jonson (11, p. 244) has developed the form-point method, which has been successfully used in Sweden. Behre (1, p. 741) reports its successful use on western yellow pine in Idaho. Wright (31, p. 81) finds approximately the same correlation between the height of the center of crown length and form quotient which he finds between form point and form quotient. Gevorkiantz and Hosley (5, p. 24) have found that by means of an equation involving the relative length of crown free from live branches and crown index ²⁷ the form quotient for white pine can be computed with a considerable degree of accuracy. Nyblom (19, p. 57) states that the form class seems to be comparatively independent of the relative length of crown, except for the extreme form classes. Heijbel (8, p. 141), working with Scotch pine, finds that in all form classes the crown ratio varies within the limits given for the entire stand, and evidently is quite independent of the form quotient. Mattsson (17, p. 228) concludes that in fully stocked stands of Scotch pine the form class increases with age. Accordingly, a table giving the average form class at various ages for stands of given densities should prove useful in form-class determination. Schumacher (27, p. 197) can find no significant correlation in second-growth redwood between form quotient and diameter breast-high, or age or site. He does, however, find a strong relationship between form quotient and the height within a given diameter class, and suggests this as a possible means of obtaining the form class.

²⁷ "Crown index" is an expression of the relationship between the diameter of crown and diameter of stem at breast height.

In view of this mass of conflicting evidence, an attempt was made to correlate relative crown length and form quotient. Measurements on crown length were available for 121 trees from Connecticut. The relative crown length was calculated for each of the 121 trees by dividing the crown length by the total height of the tree. The correlation between this ratio and the genuine form quotient is -0.40 ± 0.05 , and the standard deviation of prediction, when predicting genuine form quotient from crown ratio, is ± 0.076 .²⁸ Figure 10 shows the straight line that expresses the relationship between form quotient and the ratio of crown length to total height.²⁹ It is apparent from Figure 10 and the standard deviation of prediction that crown length has a certain amount of influence upon the genuine form quotient, but not enough to use as a predicting agent in determining the form quotient for a single tree.

In the investigation of the simultaneous influence of crown length and crown width upon genuine form quotient, an equation involving these as independent variables was determined. In this instance the crown width was expressed as a ratio between the diameter of the crown in feet and the diameter of the tree in feet. The equation thus determined is

$$Q = 0.72 - 0.39x_1 + 0.0022x_2. \quad (14)$$

In this equation Q is the genuine form quotient, x_1 is the ratio between the crown length and total height, and x_2 is the ratio between crown width and diameter at breast height. When this formula is used the standard deviation of the error in predicting the form quotient is 0.077.

An attempt was made to improve this equation by the inclusion of a third independent variable, height. The resulting equation is of the form

$$Q = 0.61 - 0.40x_1 + .0030x_2 + .0019x_3, \quad (15)$$

²⁸ This standard deviation of prediction was calculated from the formula $\sigma_e = \sigma_y \sqrt{1 - r^2}$.

²⁹ The equation of this line was calculated by the relationship $y = r \frac{\sigma_y}{\sigma_x} x$. Here y is the deviation from the mean form quotient and x is the deviation from the mean ratio between crown length and total height.

where Q , x_1 , x_2 are as in equation (14), and x_3 is the total height of the tree in feet. The standard deviation of error in predicting form quotient when this formula is used is 0.076. The addition

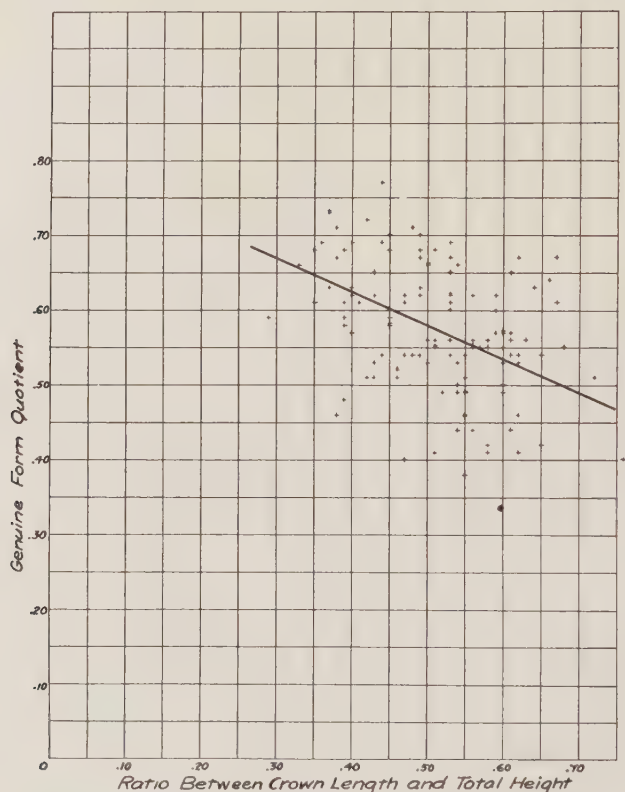


FIG. 10. Relation between relative crown length and genuine form quotient.

Equation of line is

$$Y = -0.45X + 0.80$$

Y = genuine form quotient

X = relative crown length

of height as an independent variable does not appreciably increase the accuracy of the predicting equation.

The standard deviations of the two equations and the corre-

lation coefficient show that the more complicated formulae are no more accurate instruments for the prediction of the form quotient than is the ratio between crown length and total height of tree. The smallness of the coefficients of the x_2 and x_3 terms in equations (14) and (15) supports this statement. This analysis indicates that the approximate genuine form quotient can be predicted when only the ratio between crown length and total height of the tree is known, and that in 68 per cent of such predictions the error in the predicted genuine form quotient will not be greater than 0.076. Since genuine form classes comprise, in this study, five form class units, such a predicted form quotient may be expected to be in error by an entire genuine form class in more than 50 per cent of the cases.

The work of Petrini (22, p. 606) on Norway spruce in Sweden shows a standard deviation of 5.40 form class units³⁰ when computing form class by means of the form point. This standard deviation is based on a sample of 104 trees. He also finds (22, p. 614) the correlation coefficient between form point and form quotient to be 0.351 ± 0.058 . The white oak data do not show so small a standard deviation of errors of prediction as spruce data do.

Common practice has assumed that within any stand the mean form class of the stand pictures with a relative amount of accuracy the form class of the stand. Since the standard deviation of the mean varies inversely as the square root of the number of observations, the accuracy of form class determination of a stand can be increased by increasing the number of trees upon which the average form class is based.

If one assumes the average form class of the stand to be obtainable to a reasonable degree of accuracy, there remains the variability of form of white oak as shown in Table III (p. 298). In addition to this variability there is the further consideration that the span of genuine form classes from 0.50 to 0.70 inclusive, embracing 85 per cent of the total number of trees, shows a difference of only six or seven hundredths in form factor. This means that the average variation in volume from one form class

³⁰ Multiply Petrini's standard deviation by 0.01 for purposes of comparison with the standard deviations given on pages 319 and 321.

to the adjacent form class above or below is about 2 per cent. When one considers jointly the slight variation in form factor and the high standard deviation of the ratios from which the form factors were computed, form class classification is not justified for white oak.

SUMMARY

In many instances the method of using the average of ratios will give the same results as the ratio of average diameters. However, before conclusions in studies of comparative form are drawn, care must be taken that the small form variations that may be thought to be significant are not, in reality, due to the method of securing the average ratios.

The genuine form quotient can be used to separate white oak trees into genuine form classes, but the variation of stem form from tree to tree within any genuine form class is large. The breaking-point system of classification is no more a satisfactory criterion of form than is the genuine form quotient.

In the lowest three tenths of the stem of white oak taper appears to be practically independent of genuine form class. This may be due in part to the effect of root swell, and in part to the effect of the base of crown on the stem form.

The equation of Behre does not represent the form of white oak.

Form variation appears to be independent of crown class, age and diameter at breast height. There is a tendency toward fuller form in the larger height classes, and also in the shorter-crowned trees.

Geographic distribution seems to affect the stem form slightly, but from this study it is doubtful whether such variation will affect materially the volume determination of tree stems.

Root swell is present at breast height on trees as small as the three-inch diameter class, and for larger diameter classes the amount of swelling increases. It is doubtful whether appreciable swelling reaches two tenths of the stem's height above breast height, but it frequently reaches one tenth of the height.

The bark percentage increases from breast height to the tip in trees of all diameter classes, and the breast-height bark percentage decreases with an increase in diameter class.

The genuine form-class system of classifying trees for more accurate volume determinations is not thought to be suitable or practicable for white oak, because of (1) the difficulty of accurately and simply determining the genuine form class, (2) the variability of taper on individual trees within any genuine form class, and (3) the slight variation in form factor that is found from genuine form class to genuine form class.

CONCLUSION

In an estimate of the wood volume on an area comprising a number of thousands of acres the error introduced by form variation should be smaller than a similar estimate on a small area. Table VI on page 302 indicates that the span of average form factors for various form classes is about 0.16. This means that the volumes of the lowest and highest form classes differ by about 16 per cent. The use of a single-volume table, constructed to fit the average form of these data, will be expected to introduce an error considerably smaller than 16 per cent. For estimation of volumes on larger tracts this degree of accuracy may be all that can be expected. For research purposes, however, where accurate volume determinations on small areas are required, this amount of error is large. Where the number of trees per plot is small, variations above and below the average are less likely to be compensating than they are in stands of thousands of trees. In consequence, accurate volume averages can be obtained only by giving attention to the individual tree stem. This can be done by estimating diameters at given heights on every tree on the sample area. From these measurements tree volumes in any desired unit of measurement can be determined with the use of appropriate tables. This procedure will involve an increase in the cost of this type of forest research which foresters at the present time may not be prepared to meet. When the need for greater accuracy in volume determination becomes sufficiently acute, precise methods of determination will of necessity be adopted.

APPENDIX A

The development of formula (4), page 278, is taken from Müller (18, pp. 13 and 203).

The rotation of the curve $y^2 = px^r$ about the x -axis will generate a solid whose longitudinal section may be represented as in Figure 11.

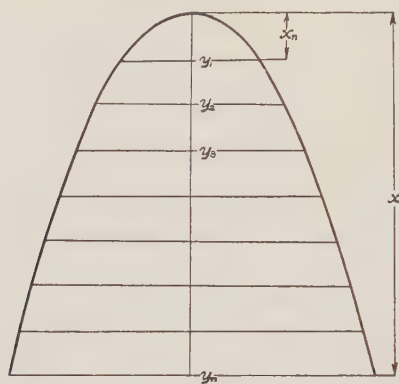


FIG. 11. Graph of parabolic curve

If x is divided into n equal units and at each unit distance from the tip a radius $y_1 \dots y_n$ is laid off

$$y_1^2 = p\left(\frac{1}{n}x\right)^r$$

$$y_2^2 = p\left(\frac{2}{n}x\right)^r$$

$$y_3^2 = p\left(\frac{3}{n}x\right)^r$$

$$\cdot \quad \cdot$$

$$\cdot \quad \cdot$$

$$\cdot \quad \cdot$$

$$y_n^2 = p\left(\frac{n}{n}x\right)^r \cdot$$

By proportion

$$\frac{y_1^2}{y_n^2} = \frac{p\left(\frac{1}{n}x\right)^r}{p\left(\frac{n}{n}x\right)^r} = \frac{1^r}{n^r}$$

$$\frac{y_2^2}{y_n^2} = \frac{2^r}{n^r}$$

$$\begin{array}{ccc} \cdot & \cdot & \\ \cdot & \cdot & \\ \cdot & \cdot & \end{array}$$

$$\frac{y_n^2}{y_n^2} = \frac{n^r}{n^r}.$$

Transposing the denominator of the left member, we have

$$y_1^2 = \left(\frac{1}{n}\right)^r y_n^2$$

$$y_2^2 = \left(\frac{2}{n}\right)^r y_n^2$$

$$\begin{array}{ccc} \cdot & \cdot & \\ \cdot & \cdot & \\ \cdot & \cdot & \end{array}$$

$$y_n^2 = \left(\frac{n}{n}\right)^r y_n^2.$$

If one considers y as a radius of a cylinder of height $\frac{x}{n}$, the volume, z , of the cylinder is

$$z_1 = y_1^2 \pi \frac{x}{n} = \left(\frac{1}{n}\right)^r y_n^2 \pi \frac{x}{n} = \frac{1^r}{n^{r+1}} y_n^2 \pi x$$

$$z_2 = y_2^2 \pi \frac{x}{n} = \left(\frac{2}{n}\right)^r y_n^2 \pi \frac{x}{n} = \frac{2^r}{n^{r+1}} y_n^2 \pi x$$

$$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

$$z_n = y_n^2 \pi \frac{x}{n} = \left(\frac{n}{n}\right)^r y_n^2 \pi \frac{x}{n} = \frac{n^r}{n^{r+1}} y_n^2 \pi x.$$

The sum of these cylinders is

$$\Sigma z = \frac{1}{n^{r+1}} y_n^2 \pi x [1^r + 2^r + 3^r + \dots + n^r].$$

The bracketed term is an arithmetic series of the r^{th} order whose sum

$$S_n = \frac{n^{r+1}}{r+1} + \frac{n^r}{f_1} + \frac{n^{r-1}}{f_2} + \dots + \frac{n^2}{f_y} + \frac{n}{f_x},$$

wherein the expressions f_1, f_2 , etc., do not need to be evaluated, since they later drop from consideration as n goes to infinity. Placing the sum formula in the brackets and multiplying by

$\frac{1}{n^{r+1}}$, we obtain

$$\Sigma z = y_n^2 \pi x \left[\frac{n^{r+1}}{n^{r+1}(r+1)} + \frac{n^r}{n^{r+1}f_1} + \frac{n^{r-1}}{n^{r+1}f_2} + \dots + \frac{n}{n^{r+1}f_x} \right]$$

$$\Sigma z = y_n^2 \pi x \left[\frac{1}{r+1} + \frac{1}{nf_1} + \frac{1}{n^2f_2} + \dots + \frac{1}{n^rf_x} \right].$$

When n approaches infinity and $\frac{x}{n}$ approaches 0

$$\frac{1}{nf_1} = \frac{1}{n^2f_2} = \frac{1}{n^rf_x} = 0,$$

and the volume of the rotation solid is

$$V = \frac{1}{r+1} y_n^2 \pi x.^{31}$$

If we substitute for the radius y the base diameter d_o , and for x the length, l , the equation gives the volume

$$V = \frac{1}{r+1} \cdot \frac{\pi}{4} d_o^2 l.$$

In forest problems the d_o is not usually measured at the base of the tree, but at some distance a above it.

$$\left(\frac{d_o}{2}\right)^2 = pl^r \qquad \left(\frac{d_a}{2}\right)^2 = p(l-a)^r.$$

³¹ This formula can be derived very easily by methods of calculus. Presumably Müller wished to make the derivation clear to students unfamiliar with calculus.

By division

$$\frac{d_o^2}{d_a^2} = \left(\frac{l}{l-a} \right)^r$$

and

$$d_o^2 = d_a^2 \left(\frac{l}{l-a} \right)^r.$$

Substituting the value above for d_o^2 the equation for volume becomes

$$V = \frac{1}{r+1} \cdot \frac{\pi}{4} d_a^2 \left(\frac{l}{l-a} \right)^r l.$$

Since the form factor is the ratio between the volume of a tree and the volume of a cylinder having the same base diameter and height, it is expressed

$$f = \frac{\frac{1}{r+1} \cdot \frac{\pi}{4} d_a^2 \left(\frac{l}{l-a} \right)^r l}{\frac{\pi}{4} d_a^2 l}$$

$$f = \frac{1}{r+1} \left(\frac{l}{l-a} \right)^r.$$

APPENDIX B

This example of the calculation of the constants for Höjer's equation for form class 0.70 is translated directly from Jonson's work (9, p. 300).

If we substitute in Höjer's equation

$$\frac{d}{D} = C \log \frac{c+l}{c},$$

known values for d , D and l , we obtain two new equations

$$\frac{70}{100} = C \log \frac{c+50}{c}$$

and

$$\frac{100}{100} = C \log \frac{c+100}{c}.$$

If we divide the upper equation by the lower, C is eliminated, and the result can be reduced to

$$0.70 \log (c + 100) = \log (c + 50) + (0.70 - 1) \log c.$$

From this equation c cannot be directly calculated, but by a trial and error process the value, $c = 19.78$, is found to satisfy the equation. If we substitute this value of c into the lower of the first two equations we obtain

$$\frac{100}{100} = C \log \frac{19.78 + 100}{19.78},$$

from which C is found to equal 1.28.

By the substitution of these known values of C and c and any measured value of D , one can calculate the diameter d at any desired height for all trees occurring in form class 0.70. This statement is made on the assumption that the species in question tapers in conformity with the curve of Höjer's equation.

APPENDIX C

The following is taken from Petterson's work (24, p. 39). The equation $y = \log x$ can be shown to be equivalent to Höjer's equation as given on page 278. In the equation $y = \log x$, as it is applied to tree stems, y is the ratio between any diameter and the base diameter, and x is the distance from the tree tip.

The total height of the tree can be represented by x_b .

$$\frac{d}{D} = \frac{\log x}{\log x_b} = \log x \frac{1}{\log x_b}. \quad (\text{a})$$

If we let

$$\frac{1}{\log x_b} = C$$

$$\frac{d}{D} = C \log x. \quad (\text{x})$$

In any coördinate system the abscissa of a point on the x -axis is the distance of that point from the origin expressed in some unit.

Assume that the unit of measurement in Figure 6, page 283, is a meters, and that a given point x on the stem lies z meters from the tip. The distance of the point from the origin is $(a + z)$ meters, or $\frac{a + z}{a}$ measuring units. Therefore, the abscissa

$$x = 1 + \frac{z}{a}.$$

Let us assume that z is l per cent of the tree's absolute length L and that a is c per cent of L . Therefore,

$$\frac{z}{a} = \frac{l}{c}.$$

If we substitute this in

$$x = 1 + \frac{z}{a} = 1 + \frac{l}{c} = \frac{c + l}{c}.$$

Substituting this value of x in equation (x) above we get

$$\frac{d}{D} = C \log \frac{c + l}{c},$$

which is Höjer's equation.

In equation (a) C equals $\frac{1}{\log x_b}$ and x_b is the base abscissa, which in Jonson's application of Höjer's equation is $\frac{c + 100}{c}$. Therefore,

$$C = \frac{1}{\log \frac{c + 100}{c}}.$$

From the relation $x_b = \frac{c + 100}{c}$ may be obtained the value for c :

$$c = \frac{100}{x_b - 1}.$$

With the help of this relationship for c Petterson calculated c for six form classes, namely, 0.55 to 0.80. In order to accomplish this he successively laid the tree's base at the points $x = 1.1, 1.2, 1.3$, etc., to the value $x = 5.0$, and thereafter at the points $x = 6, 7, 8$, etc., to the value $x = 30$. The middle diameter corresponding

to each value of x_b lies at the point $\frac{x_b + 1}{2}$. For each investigated value of x_b the form class was determined according to equation (a) by dividing $\log^{32} \frac{x_b + 1}{2}$ by $\log x_b$.

These form classes were plotted as ordinates over the x_b values as abscissae, and a curve was passed through the resulting points. From this curve the value of x_b that corresponds to each form class may be read off. The values of x_b corresponding to form classes 0.55, 0.60, etc., to form class 0.80 were determined and tabulated.

If the diameter at one tenth of the distance from the tip to breast height for form class 0.70 is desired, the procedure is as follows: From the tabulated values of x_b form class 0.70 is seen to have an x_b of 6.056. The tree's length above breast height is $6.056 - 1.0 = 5.056$ measuring units. One tenth of this length is 0.5056 measuring unit, and the desired diameter lies at the point $x = (1 + 0.5056)$. Therefore, $\log x = \log 1.5056$, which equals 0.17771. $\log x_b = \log 6.056 = 0.78219$. Therefore, the diameter at one tenth from the tip is $\frac{0.17771}{0.78219} = 0.227$ of the breast-high diameter. Other diameters for the several form classes may be computed similarly.

³² The log before this term was omitted in Petterson's text, but this must have been a typographical error.

APPENDIX D

Heijbel (8, p. 144) begins the development of his tangent equation with the simple form

$$x = \tan y. \quad (b)$$

In order to eliminate ambiguity he considers the portion of the curve lying between $y = \pm 90^\circ$. The curve has a point of inflec-

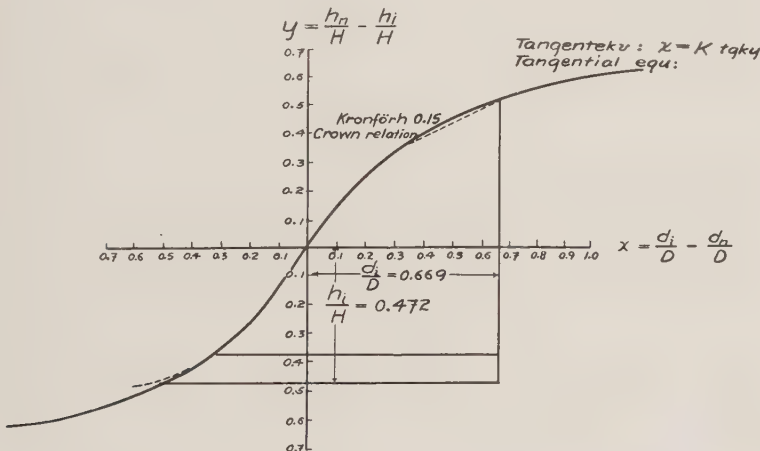


FIG. 12. Curve of Heijbel's equation. Taken from Heijbel

tion at the origin and is symmetric in the first and third quadrants. From Figure 12 it is seen that x represents the diameter quotient

$$\frac{d_n}{D} \text{ and } y \text{ represents the height quotient } \frac{h_n}{H}.$$

Since the inflection point lies at the origin, the y values read on the curve give the difference between the height quotient and the

$$\text{height quotient of the point of inflection } \frac{h_i}{H},$$

$$y = \frac{h_n}{H} - \frac{h_i}{H}.$$

Similarly, the x value gives the difference between the diameter quotient of the inflection point and the real diameter quotient:

$$x = \frac{d_i}{D} - \frac{d_n}{D}.$$

Equation (b) may then be written

$$\frac{d_n}{D} = \frac{d_i}{D} - \tan \left[\frac{h_n}{H} - \frac{h_i}{H} \right]. \quad (c)$$

In order to make possible the regulation of the necessary conditions in the x and y members, two constants, K and k , are introduced. The equation then reads

$$\frac{d_n}{D} = \frac{d_i}{D} - K \tan \left[k \left(\frac{h_n}{H} - \frac{h_i}{H} \right) \right]. \quad (d)$$

In this equation d_n is the diameter at any height h_n , d_i is the diameter at the point of inflection, and h_i is its height. D is the diameter at ten per cent of the height, and H is the total height of the tree. K and k are constants that vary with the form class and relative crown length. All heights are reckoned from the stump.

Within the crown portion of the stem the actual diameter ratios of trees deviate in a negative direction from the curve of equation (d). Heijbel likens this deviation to the difference between a second-degree and a third-degree parabola, thus,

$$\begin{aligned} y &= ax^2 \\ y_1 &= x^3 \\ y - y_1 &= ax^2 - x^3 = \delta = x^2(a - x). \end{aligned} \quad (e)$$

In this equation δ denotes the difference between the two parabolas. The symbol x corresponds to the height quotient,³³ reckoned from the base of the crown, at which point the two curves are tangent to each other. The point of tangency is indicated by h_a , and is given in percentage of the height of the tree. Accord-

³³ It is realized that Heijbel allows x to represent height in one equation and diameter in another, but since this material is almost a literal translation, the author's symbols are retained.

ingly $\frac{h_a}{H}$ is the beginning of the crown. The symbol a is a constant that gives the scale of the y coördinate for the parabola of the second degree when the corresponding scale for the parabola of the third degree is unity. For the reduction of the differences the constant C is introduced. By employing the symbols used in the preceding equation

$$\delta = C \left(\frac{h_n}{H} - \frac{h_a}{H} \right)^2 \left[a - \left(\frac{h_n}{H} - \frac{h_a}{H} \right) \right]. \quad (f)$$

In this equation a has the special property not only of denoting the magnitude of the scale in the parabola of second degree, but also the value of the x coördinate at the intersection point of the two curves. In the tree this represents the relative length of crown, or crown ratio.

It can easily be shown from equation (f) that a is equal to the crown ratio. At the tip the two curves coincide and δ is zero. In this case $\frac{h_n}{H} - \frac{h_a}{H}$ is equal to the crown ratio B . If we assume δ to be 0 in equation (f)

$$0 = a - \left(\frac{h_n}{H} - \frac{h_a}{H} \right).$$

But

$$B = \frac{h_n}{H} - \frac{h_a}{H}.$$

Therefore

$$a = B.$$

Since the total height of the tree is 1, the crown ratio can be obtained from the base of the crown by means of the following identity: $B = 1 - \frac{h_a}{H} = a$. Equation (f) may then be written

$$\begin{aligned} \delta &= C \left(\frac{h_n}{H} - \frac{h_a}{H} \right)^2 \left(1 - \frac{h_a}{H} - \frac{h_n}{H} + \frac{h_a}{H} \right) \\ \delta &= C \left(\frac{h_n}{H} - \frac{h_a}{H} \right)^2 \left(1 - \frac{h_n}{H} \right). \end{aligned} \quad (g)$$

C is a constant for any given crown ratio, and seems to be unaffected by the form class and crown class of the main curve.

The stem portion from the stump to ten per cent of the height is represented by a power curve of the form

$$y = r(x)^{\frac{1}{3}} \quad (h)$$

Here y is the diameter quotient and x the height quotient. The diameter quotient of the stump diameter, $\frac{d_o}{D}$, corresponds to a height quotient of zero, and the base diameter, D , is measured at ten per cent of the tree's height above the stump. After substituting the symbols previously used for the other equations, equation (h) may be written

$$\frac{d_o}{D} - \frac{d_n}{D} = r \left(\frac{h_n}{H} \right)^{\frac{1}{3}},$$

or

$$\frac{d_n}{D} = \frac{d_o}{D} - r \left(\frac{h_n}{H} \right)^{\frac{1}{3}}. \quad (i)$$

The symbol r may be determined by appropriate computations, and from equation (i) diameter quotients for points lying between the stump and ten per cent of the tree's height may be calculated.

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Med. Förs. = Meddelanden från Statens Skogsförsöksanstalt, Stockholm.

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